A TOOLBOX WITH DERIVE

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Abstract: An analysis of some of the characteristics of the European Higher Education Area (EHEA), its difference in relation to the current University teaching system and the role that new technologies might play in this new scenario has been performed. This paper suggests a new possibility in use of technologies: The design of a “toolbox” with DERIVE instructions about topics in a usual Calculus course.

Introduction

The implementation of the EHEA (see [1, 2]) implies new teaching methods taking into account that the students now are the centre of the learning process. The role of teachers changes and they must be able to guide their students’ work (see [6, 7, 8, 9]).

Teachers are currently being required to change the traditional teaching model in order to adapt to “learning based on competences”. It is necessary to define the competences to be acquired by the students after attending a course on a certain subject, and to design the activities according these competences.

Such change may fail if a considerable amount of effort, imagination, common sense, and hope is not devoted to it. The system inertia and the difficulty involved in designing effective activities must be borne in mind, since teachers generally have extensive experience in preparing expository lessons, with more or less of students, and although we have worked hard to find the best way of introducing and presenting concepts and results, we do not have any experience in guiding the search.

The new teaching model implies more autonomous work by students. To see this it is merely necessary to analyze the new structure of studies, which are articulated in 60 yearly ECTS (European Credit Transfer System) credits, where each ECTS credit must reflect between 25 and 30 hours of the students’ “overall work”, only 10 to 15 of which must necessarily involve classroom attendance.

This is why teachers find it challenging to design a mathematical course for engineering students (for example a course on Calculus of One Variable or a course on Linear Algebra) taking these determining factors into account.

In our opinion we are doomed to design mathematical courses in which *magisterial* lessons (theory and problems solved in detail by the teacher), *practical workshops* (problem solved and Mathematical laboratories based in a CAS), and *tutorial activities* must be blended so that students can acquire the required competences.

Laboratory classes must be designed with clear goals. Our proposal is that Computer Algebra System (CAS) could help in the automatic performance of certain tasks involved in the problem-solving process. For this, it would sometimes be necessary to use certain functions or commands which might already be integrated to the system or which might have been prepared by the teacher, or even created by the students themselves.

Working in a more autonomous way allows students to access computing technologies outside conventional training, which is why it is essential that they acquire the skills to make optimum use of them. The advantages of CAS must be boosted: visualization, computation facilities, the possibility of experimenting…, avoiding possibly damaging effects such as the lack of a critical attitude when considering the computer response, an inability to interpret the results, etc.

In any case, in order to be effective all the activities suggested must be designed without letting the intended goals out of sight, mainly taking into account the students at whom they are aimed.

1. A box of mathematical tools

Mathematical subjects, which are usually programmed within the first years of Engineering studies, have as their main goal the initiation of students into the language of Science and Technology and their preparation in the correct use of certain algorithms in problem solving.

Teachers often complain that outside the context of the subject (for example, in later years) students are not able to use the mathematical skills acquired during the basic years of their training. They seem to have a kind of "mental laziness" that prevents them from remembering and using what they have learnt, and in many cases they do not have a fast and effective way to find the information or the appropriate methods either.

The use of technology in the classroom is increasing and in certain cases students are even asked to define some tools that allow them to automate certain simple tasks, such as for example the implementation of a function to calculate the tangent to a curve $y = f(x)$ at a point and subsequently use it in other problems. In general, however, when teachers suggest to students that they should define a method aimed at automating a mathematical task they merely wish to help them understand the corresponding algorithm and they do not usually expect students to design their own resources or use the implemented functions in following years.

One educational activity is to encourage students to create their own well-organized “Toolbox” for solving mathematical problems. This toolbox is no more than a file or collection of files of utilities, programmed in the characteristic programming language of a CAS (Derive, Maple, Maxima) or even on a calculator with symbolic or graphic capacities such as the TI92 or Casio ClassPad 300.

The teacher should suggest a series of essential tools, depending on the corresponding subject, that the box should contain. Students must to define the corresponding functions, test them, and add those they find appropriate. In addition, they must complete their work by writing a “brief user’s manual” for their tools.

If they have created a good toolbox, and they have also understood the algorithms well, they will then have their own resource, which they will be able to use in other subjects during the same or following years.

Before finishing with the general ideas on toolboxes it should be noted that nearly all CAS offer a very thorough toolbox and that the creation of new tools, or the modification of those already available, will only be necessary or convenient for reasons of ease of use or as a teaching strategy in order to adapt them to the user’s specific needs.

2. A toolbox of Calculus

In this section we propose a toolbox that can be done by students of Calculus in the first course of Engineering, whose reference text book may be [3 or 5].

Before present our toolbox, is necessary to make it clear that in DERIVE are implemented the most of the instructions for the study of Calculus of One Variable. Thus, the calculation of limits, derivatives, integrals or Taylor polynomials is “basic” using the DERIVE menu or instructions.

Following the strategy referred to in the previous section, a *supplementary* toolbox has been created that can be used in a *more specific way*.

With the toolbox our aim is to extend the use of the CAS, in our case DERIVE, and we therefore try to use, as far as possible, its symbolic, numerical, and graphic capacities.

Some of the utilities are “improvements” to DERIVE instructions (for example the **TANGENT2** method, where a distinction is made between differentiable and non- differentiable functions). Other tools are analogous to known instructions integrated in DERIVE, whose syntax is complicated. For this reason we prefer students to automate the algorithms according to their own criterion, so that they get a more continuous use when they have to apply these concepts in other topics. In addition, this automation “guarantees” that they have understood the corresponding concepts.

Our toolbox could be designed with several compartments, one for each section of Calculus of One Variable course:

- Complex Numbers.
- Limits and Continuity.

- Differentiability.
- Integral Calculus (including numerical integration).
- Numerical Methods (for solving nonlinear equations).
-

Below we briefly explain the tools.

2.1. Complex Numbers

The Complex Number compartment includes tools for:

- Convert a complex number to exponential form.
- Plot a complex number as a pair of real numbers.
- Compute a list with the n^{th} roots of a complex number.
- Find the n vertices of a regular polygon.

The following example allows one to calculate the n^{th} roots of a complex number and, taking advantage of the graphic capacities of DERIVE, plot a start.

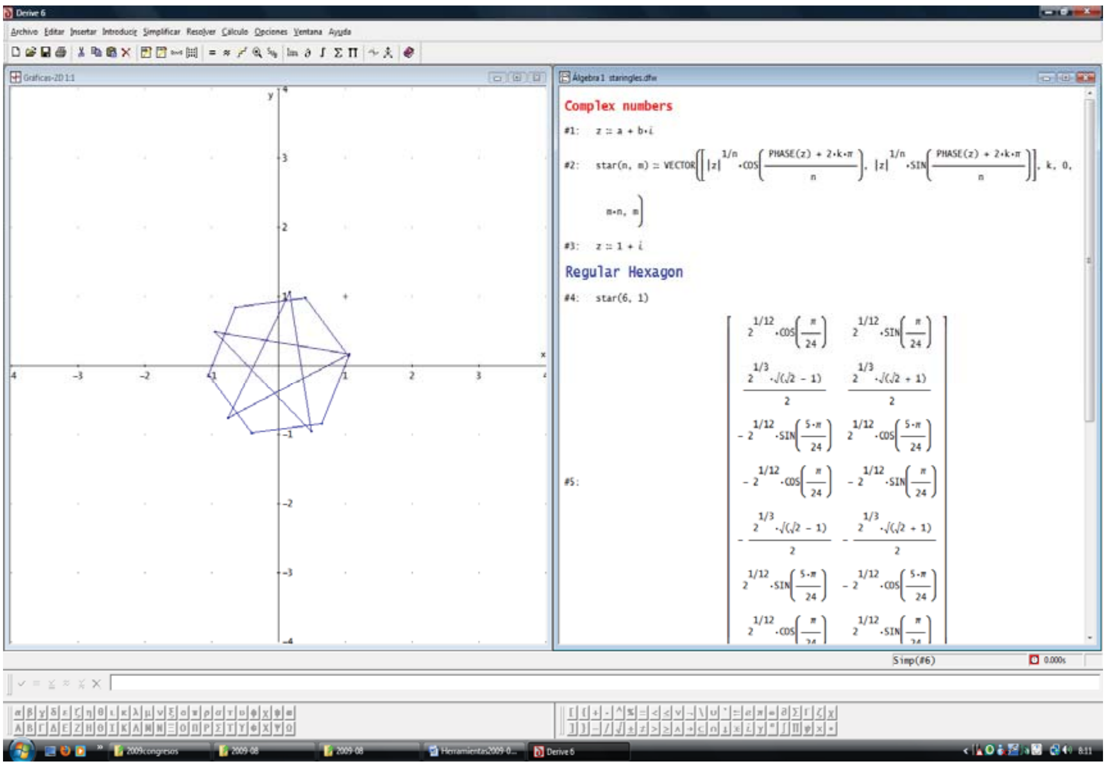


Figure 1: A regular hexagon with a six-pointed start

2.2. Differential calculus

The utilities performed allow the analysis of the continuity or discontinuity of the function at a point (studying the one sided limits, analyzing the equality of the obtained values and comparing with the value of the function at the point). The differentiability of a function at a point is studied following the same strategy.

The calculation of the tangent line has been implemented, with additional information to the **TANGENT** function of DERIVE, since it tells us when the function is not differentiable at the point (see figure 2).

```
The tangent

#1:   f(x) :=

                      f(a + h) - f(a)
#2:   DL(a) := lim   ─────────────────
               h→0-          h

                      f(a + h) - f(a)
#3:   DR(a) := lim   ─────────────────
               h→0+          h

      TANGENTE2(a) :=
        If DR(a) - DL(a) = 0 ∧ (1/DR(a))^2 > 0
#4:        y = f(a) + DR(a)·(x - a)
           "f has no derivative"

      f(x) :=
        If x ≤ 0
#5:        x
           x^2

#6:   TANGENTE2(0)

#7:                                          f has no derivative

With the DERIVE instruction TANGENT the result is not correct

#8:   TANGENT(f(x), x, 0)

#9:                                                   x
```

Figure 2: `TANGENT` and `TANGENT2` functions

Taking advantage of DERIVE graphical capacities and of the structure of the IF instruction, the monotonicity and concavity or convexity of a sufficiently differentiable function can be analyzed.

For example, with the instruction **`CRECE(x):= IF(F'(x)>0,F(x))`** it is possible to represent the curve $y = F(x)$ in the intervals where F is increasing.

The graph of figure 3 has been obtained by applying the instruction **`CRECE(x)`** with $F(x) = x^3 - x$

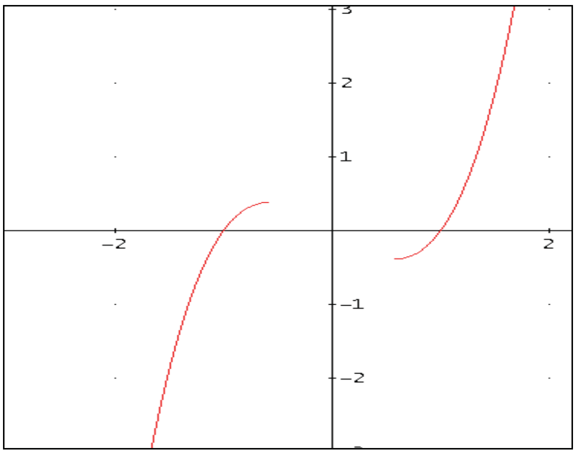


Figure 3: Intervals where $F(x) = x^3 - x$ is increasing

Other functions about Differential Calculus in the toolbox may be:

`ROLLE_POINT` for finding a point according Rolle's Theorem hypotheses.

`LAGRANGE_POINT` for finding a point according Lagrange's Theorem hypotheses.

These functions can work in exact or approximate way, because it is necessary solving equations.

2.3. Integral calculus

In order to introduce the Riemann integral we can use (see [4]) the DERIVE instructions: **AreaUnder-Curve** and **LEFT_RIEMANN**. Furthermore we have included in the toolbox the **DER_RIEMANN** for calculating the sum for right rectangles associated to Riemann sums. We also have implemented the **rect_izq** and **rect_der** functions for plotting the left and right rectangles associated to Riemann sums

We promote our students to define simple instructions for computing lengths, areas and volumes with a syntax more recognizable that **POLAR_ARC_LENGTH, PARA_ARC_LENGTH, POLAR_AREA, VOLUME_OF_REVOLUTION, AREA_OF_REVOLUTION,** etc.

For instance, to calculate the length of an arc of the curve $y = f(x)$, it is possible to define the function **LEXP** and theoretically find the length of any arc of curve.
The figure 4 shows the DERIVE implementation and the calculation of the parabola's length $y = x^2$ between the abscissas 1 and 2.

#1: f(x) :=

#2: $\mathrm{LEXP}(a, b) := \int_a^b \sqrt{(1 + f'(x)^2)}\, dx$

The parabola length

#3: $f(x) := x^2$

#4: LEXP(1, 2)

#5: $\dfrac{\mathrm{LN}(\sqrt{85} - 2\cdot\sqrt{17} + 4\cdot\sqrt{5} - 8)}{4} + \sqrt{17} - \dfrac{\sqrt{5}}{2}$

And the approximate value

#6: 3.167840904

Figure 4: The length of an arc of curve

Finally, students can define tools for numerical integration using the Composite-Trapezoidal rule and Composite-Simpson rule (see figure 5).

#1: f(x) :=

#2: $$\mathrm{TRAP}(a, b, n) := \frac{b-a}{n}\cdot\left(\frac{f(a)+f(b)}{2} + \sum_{i=1}^{n-1} f\left(a + \frac{i\cdot(b-a)}{n}\right)\right)$$

#3: $$\mathrm{SIMPSON}(a, b, n) := \frac{b-a}{3\cdot n}\cdot\left(f(a) + f(b) + 4\cdot\sum_{i=1}^{n/2} f\left(a + \frac{(2\cdot i - 1)\cdot(b-a)}{n}\right) + 2\cdot\sum_{i=1}^{n/2-1} f\left(a + \frac{2\cdot i\cdot(b-a)}{n}\right)\right)$$

#4: $$f(x) := \frac{x\cdot e^{-2\cdot x}}{x^2 + 4}$$

#5: $$\int_{-1}^{1} f(x)\,dx$$

#6: $$\int_{-1}^{1} \frac{x\cdot e^{-2\cdot x}}{x^2 + 4}\,dx$$

#7: [TRAP(-1, 1, 10), SIMPSON(-1, 1, 10)]

#8: [-0.4342879366, -0.4215187544]

Figure 5: Numerical Integration

2.4. Numerical calculus for solving nonlinear equations

The methods usually taught to students for solving equations are: Bisection, Newton and fixed-point. DERIVE has the **NEWTON** and **FIXED_POINT** algorithms integrated, so we can propose that students define a function to implement the bisection method. An algorithm for this method could as follows:

```
H(a, b) :=
  If F((a + b)/2)·F(a) < 0
     [a, (a + b)/2]
     If F((a + b)/2)·F(b) < 0
        [(a + b)/2, b]
        [(a + b)/2, (a + b)/2]

BISECC(a, b, n) := ITERATES(H(v , v ), v, [a, b], n)
                               1   2
```

Figure 6: Bisection Method

The instruction **BISSEC(a,b,n)** (see figure 6) computes the interval obtained after n iterations of Bisection method, to solve the equation $f(x)=0$ in the interval $[a,b]$.

3. Using the toolbox to solve technical problems

Students are encouraged to use the toolbox to solve technical problems. Here we set out one example, taken from an Environmental Sciences exam that has been used by our engineering students.

Specifically, the equation to be solved (which appears together with its solution in the figure 6) provides the height of the chimney of a thermoelectrical station in a rural area with the required quality standards.

"

Numerical methods

```
#1:   F(x) :=

#2:   PUNTO_FIJO(x0, n) := ITERATES(F(x), x, x0, n)

#3:   F(x) :=
```

#4: $\mathrm{NW}(a, n) := \mathrm{ITERATES}\left(x - \dfrac{F(x)}{F'(x)}, x, a, n\right)$

```
      H(a, b) :=
        If F((a + b)/2)·F(a) < 0
           [a, (a + b)/2]
#5:        If F((a + b)/2)·F(b) < 0
              [(a + b)/2, b]
              [(a + b)/2, (a + b)/2]
```

#6: $\mathrm{BISECC}(a, b, n) := \mathrm{ITERATES}(H(v_1, v_2), v, [a, b], n)$

Taken from an exam in my University (April 2009).
Topic: Environmental techniques

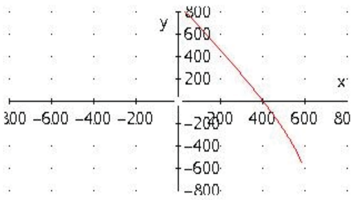
The goal: To find the minimum height of a chimney of a thermoelectric central in a rural area to be concordant with the quality environmental standards. "

"

```
The equation to be solved for finding the height:
H=1.6*575.29^(1/3)(10(588-H)^(2/3))/5
```

#7: $F(x) := \dfrac{1.6\cdot 575.29^{1/3}\cdot(10\cdot(588 - x))^{2/3}}{5} - x$

#8: $F(H) := \dfrac{1.6\cdot 575.29^{1/3}\cdot(10\cdot(588 - H))^{2/3}}{5} - H$



Sketch the graph to approach the initial values of bisection method:

#9: BISECC(200, 500, 10)

#10:

200	500
350	500
350	425
387.5	425
387.5	406.25
396.875	406.25
401.5625	406.25
401.5625	403.90625
401.5625	402.734375
402.1484375	402.734375
402.1484375	402.4414062

Figure 7: The height of a chimney

For engineering purposes the chimney height is around 400 meters.

Conclusion

The use of a toolbox adapted to each student's individual needs containing instructions that the students themselves find useful must, we believe, surely reinforce the learning process, since students participate actively and the use of the CAS is not limited to what is sometimes "blind" use of the CAS instructions.

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