TOWARDS THE EUROPEAN HIGHER EDUCATION AREA:
A BALANCED USE OF THE CAS.

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Abstract

The European Higher Education Area (EHEA) implies, at least in Spain, a change in the traditional teaching of mathematics in Engineering Schools. In some cases, the new teaching based on the acquisition of competences involves an authentic methodological revolution in which a tutorial activity and the use of new techniques, including Computer Algebra Systems (CAS), have to play an important role.

In this paper we will show how to develop within this educational frame a teaching strategy that will permit to tackle in a global and multidisciplinary form the complete resolution of different Engineering problems that can be modelled in mathematical terms, using this context to get the students to acquire an effective command of the use of the mathematical method in the resolution of Engineering problems. To carry out this strategy implies for each problem we tackle, to establish concrete objectives, the exposition of the involved mathematical elements, the search for the solution from the Engineering point of view, the choice of the adequate methods, the introduction of possible techniques of solution, exact and approximate, the qualitative and quantitative study of the problem and so on.

Some reflections will be offered on the advantages and inconveniences of this strategy and on the other part, by means of different examples the opportunity and intensity of the use of the different CAS will be analysed as support of the student's learning and competences acquiring process. Some examples related to electric and electronic circuits will be developed at length since its study covers a wide range of mathematical techniques such as systems of linear equations, solution of ordinary differential equations, Laplace transforms, Fourier analysis, and control theory, etc.

Introduction

Convergence in the structure of the different studies in the European Higher Education Area (EHEA), derived from the Declaration of Bologna, is the huge task awaiting the different European university systems. The reform affects not only the structure of University studies but, also, it elicits a reflection within the European environment as regards the convergence of mathematics subjects and, more importantly, the way in which mathematics is to be taught. It is no longer possible to the maintain the same style of mathematics teaching as that imparted 50 years ago, and the technological revolution that has emerged in recent decades should be made visible in the classrooms.

Along this paper, we present our proposal for teaching, the aim being to provide some insights into the new challenges arising in mathematics teaching in Spanish Engineering Schools.

1. The European Higher Education Area

With a view to contextualising this new paradigm of European university teaching, we should first list some characteristics of the EHEA currently under design:

-A teaching offer based on competences.

-A diversified teaching offer (theoretical, practical, directed academic activities, independent work by students, etc).

-A preoccupation with the overall work of students in contraposition to the current system, in which the only measure in some European countries, such as Spain, is the number of class hours. Such work should have a maximum annual volume estimated at 60 European credits (European credit Transfer System, ECTS). At an initial estimate, it may be considered that 1 ECTS credit is equivalent to 30 hours of student work, including class attendance, laboratory work, workshops, individual and collective tutorials, individual or group work, and assessment.

-A preponderance of student learning over the face-to-face teaching given by the instructor.

-A duration of University studies more akin to reality.

The demands of the teaching staff, coming from the policy of harmonisation at European level of higher education, are therefore:

-That instructors should teach contents with value in the job market (competences).

-That instructors should teach in a different way (methodological innovation).

-That students and their needs should be the basic criterion for the programming and teaching of the various subjects (the students as referent).

All this should involve greater dedication by instructors to the student learning process; this should include teaching hours, time devoted to organisation, the preparation of materials, the proposal of tasks, the orientation and management of such tasks, and, in general supervision of the whole of the learning process.

2. A new scenario for teaching practices

According to the characteristics of the EHEA and the demands of the teaching staff mentioned above, certain changes, sometimes drastic, are being imposed on our usual teaching practices. The change demanded affects not only the contents, derived from a less initial understanding and a less mathematical background acquired in High School (as seen from different reports coming out of Europe) but, also, it is necessary to tackle a methodological change that will allow us to cope with the new learning requirements of students. Use of the vast potential offered by the new technologies as applied to teaching activities may lead to the need to reappraise the organisation of different teaching teams, adapting size to the possibilities of the different classrooms.

Finally, we must change the assessment process, finding mechanisms that will evaluate the whole learning process. Again, the new technologies are the key element in this, allowing us to put into practice properly designed self-assessment processes, tutored practices, etc.

This completely new situation means that instructors must pay special attention when programming the following elements:

1. The competences that the student will acquire independently within the context of the whole offer of competences taught in the subject in question.
2. The academic scenarios where the instructor wishes to teach, as referred to specialised teaching facilities (IT, laboratories, audiovisual facilities, etc), libraries and other document facilities, as long as students have the possibility of developing their skills there independently. There is also, the issue of appropriate use of e-learning and b-learning to help in teaching.
3. The work schedule to be followed by students for the acquisition of such competences, specifying the stages involved in the whole learning process, the tasks to be performed by the students and by the instructor in each of the stages, and the estimated duration for each (proposal of the number of hours and schedule).
4. The tutorial-supervision tandem that the instructor has generated to control the acquisition of competences by students. This implies that we must specify at least three elements: the format of the supervision (face-to-face, distance learning, or mixed); the amount of time available to students for entering into contact with their instructors to explain what they have done (number of times they must attend tutorials, access web-sites or communicate via e-mail with their instructors, and when this is actually appropriate), and the elements and requisites for supervision (what the students should present and how, written documents, etc).
5. The system of assessment of the competences acquired by the students on their own. In this sense, we must design the assessment criteria (the results expected in each stage and the overall results of the process), the relative evidence of each criterion (data

collected, reports generated, etc) that can be evaluated, the assessment tools, and the evaluation schedule.

The subject program should therefore stipulate all these elements in the offer of tutored learning carried out by the instructor.

To a greater or lesser extent, instructors will have to provide students with the pertinent material, give lectures, set supervised work or problems, carry out tests or controls, use the new technologies, etc., to generate the number of hours corresponding to the assigned credits, which must also be controlled effectively.

Within this panorama, it behoves us to highlight the important that all non-face-to-face teaching strategies will have in the new academic configuration. In this sense, as occurred with the introduction of the different IT systems of mathematical calculation, the meaning of the term e-learning is ambiguous and admits several interpretations. From our point of view, in order to speak of e-learning in our field it is necessary that the proposal of non-face-to-face learning should not be limited to the availability of class material on the Internet. We need interactive material, material with different degrees of complexity that will enable self-evaluation and allow instructors to measure the knowledge acquired with such a mechanism.

In recent years, most Universities have been offering their teaching staff on-line teaching systems that allow material to be transferred, participation in chats and forums, the handing-in of student tasks, communication with the student body, self-evaluation, etc.

In sum, e-learning (in the broadest sense of the term) should be considered as yet another channel in the student learning process that may sometimes be a complement to and at other times an alternative to “traditional” teaching methods. This channel will acquire increasing importance since use of the Internet will also become increasingly relevant when programming academic activity. In the case of making the use of virtual material compatible with face-to-face teaching, we shall be facing a new concept that, in our opinion, is due to play a crucial role in forthcoming years: the so-called b-learning, an increasingly popular concept in teaching circles.

3. What shall we do with Maths?

Mathematical contents are included in all graduates programs in Engineering Schools. It is clear that mathematics will continue to fulfil a dual function within the new University framework. On one hand, mathematics will still offer a powerful training tool and, on the other, it will continue to be an essential support for other academic disciplines. Accordingly, the need for basic mathematic knowledge within the so-called “acquisition of competences” will remain crucial. Additionally, the necessary methodological changes are of special importance in mathematics subjects. It should not be forgotten that, at least in Spain, we shall be passing from an eminently face-to-face mode of teaching to a mixed mode, in which students, under the supervision of the instructor, will have to acquire new knowledge through their own efforts.

First, we must provide the objectives for mathematical learning. Accordingly, all Engineering graduates will have knowledge of, an understanding of, and the ability to use mathematical methods and techniques appropriate to their programme.

Common ground for almost all programs includes:

-Calculus in one and several real variables

-Linear Algebra

-Ordinary Differential Equations

The skills we consider may be expected are:

-The ability to model a situation mathematically

-The ability to solve problems using mathematics tools

It is clear that solving problems nowdays should include their numerical and computational resolution.

The skills are developed progressively, through the practice of many subjects.

We propose, and are indeed now putting into practice, a mixed model of teaching, which considers the generation of support internet material, to facilitate students' work. There have been many experiments in this sense, such as the *Xmath* project [2],[3],[12], in which a course of Calculus I has been designed with a modal structure. Each mode contains theory, exercises, tests, problems, miscellanea, puzzles, etc, together with links to web pages though to be of interest. A pilot course (*Xmath pilot course*) has been compiled. This course has been assessed and the replies from the students (Norwegians, Slovakians, and Spaniards) have been fairly homogeneous. In general the students found the course interesting as a back-up to the subject and appreciated the different levels of depth of the material. The main criticisms were about the design of the web page and the links used in the course [11]. dMath [4] is the new European project, continuation of Xmath.

Our mathematics teaching model combines the use of electronic or e-self-contained material and face-to-face teaching. A CAS is used as a work tool by the instructor for explaining some results, and as a work tool for students for the solving of certain problems, in this case following the general criterion of using the computer as a black box to carry out routine tasks, whose assimilation is assumed to have been completed in previous stages.

Student assessment is adapted to this methodology, such that the students are even allowed to use the CAS for exams.

4. Some possibilities

4.1. Interdisciplinary projects

The more flexible distribution of study time allows students to engage in a team-work project in which they address a real problem, performing the complete task of document acquisition, problem modelling, complete problem resolution using different techniques (using the CAS where necessary), interpretation of the results, and presentation of the conclusions.

The most important innovation of this proposal is its interdisciplinary nature, since the project will measure the acquisition of competences with respect to the contents given in each subject. In this case, we develop an example that initially involves only the different mathematics subjects, although in a second phase the collaboration should be extended to instructors of specific Engineering Departments.

In this sense, the example selected consists in using, as the driver of the application of knowledge in mathematics, the different possibilities that may appear in the study of electrical circuits.

These circuits mainly contain resistors, inductors, capacitors, electromotive forces (emf). Their modelling and resolution involves, among others, the following mathematical concepts and tools:

- Complex numbers
- Systems of linear equations
- Eigenvectors and eigenvalues
- Ordinary differential equations
- Fourier series
- Numerical methods

The work is done in teams, who must plan the distribution of tasks. The aim is that the students should acquire real data, posit the mathematical modelling of problems associated with electrical circuits, use different strategies such as exact or approximate resolution, study of the behaviour of the system, qualitative analysis of the solution, etc.

The manner of presenting the conclusions of the work will depend on the number of teams and take the form of an oral defence or a suitably compiled document. The project would involve the following phases:

The simplest electrical circuits, those only containing resistors, are modelled by sets of linear equations, in application to each mesh of Kirchoff's laws. Accordingly, one starts with several examples proposed by the instructor that will allow students to recall those laws. An initial research task, then, is to re-discover the expressions that provide the resistance equivalent to resistors in series and in parallel. A later task is the search by the students for examples of circuits of increasing complexity, if necessary consulting with instructors from technical departments. Depending on the complexity of the data and on the meshes present, it will be necessary to use a CAS both for solving the problems accurately and for using the appropriate numerical methods of linear algebra. In this phase of the project, would provide the most advantaged students with an introduction to selective modal analysis.

The next phase begins with simple examples containing non-resistive elements. Thus, using complex numbers appropriately, concepts such as impedance, phase shift and resonance appear.

The modelling of this type of circuit leads in a natural way to the environment of ordinary differential equations. Again, the mission of the instructor is to posit simple examples, initially with sinusoidal electromotive forces, that help the students to recall the solution of second-order differential equations, identifying the solution of the homogeneous linear equation (transient regimen) and the particular solution to the non homogeneous ODE (steady regimen). It is also of interest that students should adequately relate the presence or not of some elements of the circuit with the order of the corresponding differential equation and the meaning of the different coefficients of the differential equation. Orientation by the instructor will allow the students to address increasingly complex problems, in which the need to resolve multi-mesh circuits appears and hence in which higher-order equations and the systems of differential equations appear. Again, the need for more complex calculations will demand suitable use of a CAS, either through exact or approximated techniques.

Another step, in a more realistic analysis, is study of such circuits assuming that electrical resistance R, the inductance L and the capacitance C vary, for example, with time and/or temperature, inevitably leading to the solution of an equation or system of differential equations of variable coefficients with the appropriate numerical techniques.

A new research task is to search for technical situations modelled by the same second-order differential equation. In this respect, the mechanical analogies (mass-spring mechanisms) emerge in a natural way, although they are not the only ones that can be studied from the mathematical point of view. Thus, mathematical modelling allows different problems in the technical environment to be related and allows mathematics to be seen as a certain “common denominator”.

A new phase of the project is the analysis of non-sinusoidal electromotive forces. One frequently encounters electrical circuits whose excitation is given by square or triangular waves with a certain period. The appropriate mathematical field for dealing with this type of circuit is that of Fourier series. In this case, it is necessary to analyze the harmonics associated with the wave in question and analyse their importance as a function of the frequency of those harmonics. The next step is for the students to find examples where the Fourier series is substituted by the Fourier transform and solve the cases using continuous or discrete techniques (discrete Fourier transform and fast Fourier transform).

The final part of the project could consist in the students searching for electrical circuits that do not form part of their usual knowledge, such as the study of certain chaotic circuits.

4.2. Solving problems

The multidisciplinary project described in the previous section is conceived for students in the last years of their degree course. However, the project could be broken up and could move the resolution of the problems involved in their syllabus to the corresponding mathematics subjects.

This less ambitious possibility has the advantage of allowing the usefulness of mathematics to be seen at times when different concepts are being introduced.

5. Conclusions

The EHEA is a new scenario teaching. Teachers must pay special attention to: The competences acquired by the students, the scenarios of teaching, the student's global work including the tutorial and independent activities and the system of assessment.

In a general sense, Mathematics must be a powerful training tool and, on the other, it will continue to be an essential support for other academic disciplines. We can profit all possibilities of mathematics, including the use of CAS, to solve real problems, with exact or approximate techniques.

The developed example, concerning electric circuits, allows offering the students the interrelation between mathematics and technical subjects.

Final remark

In an annexed folder, A60_ANNEX, appear several files (CIRCUITS.pdf, GIBBS.pdf, FOURIER.pdf, RIEMANN.pdf) with some possible students' works, developed with Maple and DERIVE, dealing with different parts of the project.

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