An example of competence-based learning: Use of Maxima in Linear Algebra for Engineers

By Ana Díaz[1], Alfonsa García[2], Agustín de la Villa[3]

[1]Dpto. de Matemática Aplicada. Universidad Nacional de Educación a Distancia, Spain, adiaz@ind.uned.es
[2]Dpto. de Matemática Aplicada, E.U. Informática, Universidad Politécnica de Madrid, Spain, alfonsa.garcia@eui.upm.es
[3]Departamento de Matemática Aplicada y Computación, ETSI (ICAI), Universidad Pontificia Comillas and Departamento de Matemática Aplicada, E.U.I.T.I, Universidad Politécnica de Madrid, Spain, avilla@upcomillas.es

This paper analyzes the role of Computer Algebra Systems (CAS) in a model of learning based on competences. The proposal is an e-learning model Linear Algebra course for Engineering, which includes the use of a CAS (Maxima) and focuses on problem solving. A reference model has been taken from the Spanish Open University.
The proper use of CAS is defined as an indicator of the generic competence: Use of Technology. Additionally, we show that using CAS could help to enhance the following generic competences: Self Learning, Planning and Organization, Communication and Writing, Mathematical and Technical Writing, Information Management and *Critical Thinking*.
A follow-up at several stages of the resolution of a problem-example shall be performed; analyzing each contribution of Maxima to the competences´ acquisition. The problem-example concerns an electronic device for a building surveillance system, with different possibilities according to the number of sides of the building and other parameters.

1 INTRODUCTION

Adaptation to the European Area of Higher Education (EAHE) implies a new teaching and learning model with active methodologies. Therefore it will be necessary to adjust all methodological resources to this new scenario. The Tuning Project proposes programs which are described in terms of specific and generic competences. A specification of competences to be acquired is necessary for the design of effective learning activities.

The concept of competence can be defined as the ability of carry out tasks or to deal with situations effectively using knowledge; skills and attitudes (see Weinert, 2001). According to Mulder et al (2006), *knowledge is captured as cognitive competence, skills as functional competence and attitudes as social competence*. Meta-competence is the overarching ability under which competence shelters (Brown, 1995). Meta-competence is described as the ability to apply competences effectively in many different situations.

The meta-competence to be achieved by engineering students of is “to be a good engineer to fit into society”. Universities propose concrete competences that need to be developed through the study of different subjects.

One of the basic courses in engineering degrees is Mathematics, which is usually distributed across various subjects. It could be said that the meta-competence associated with this matter is the ability to master the mathematical techniques that allow students to solve engineering problems. These techniques are beyond mathematical concepts. Thus, the teaching of mathematics should always be mindful of problem-solving strategies, and hence in each mathematical discipline the habit of previously discussing the use of continuous or discrete, analytical or numerical methods, should be encouraged. Furthermore, it may be appropriate to start a simplified problem-solving model by using well known techniques, such as linearizing a non-linear model, etc. The methodology of problem-based learning is closely related to the style of teaching proposed. However, problem-based learning methodology must be subject to a careful analysis in order to ensure its effectiveness. In the first year of a degree course it may be highly desirable to introduce concepts and basic training skills before proposing problems.

Here we focus on the Linear Algebra subject, proposing the use of a CAS in an e-learning model. Our reference course is carried out at the Spanish Open University with engineering students. This course is associated with the development of the following competences:

Generic competences

- G1: Self Learning.
- G2: Analysis and Synthesis.
- G3: Planning and Organization
- G4: Communication and Writing.
- G5: Mathematical and Technical Writing.
- G6: Use of Technology.
- G7: Information Management.
- G8: Critical Thinking.

Specific competences

- S1: Knowledge, understanding and use of the principles of basic training in Linear Algebra.
- S2: The ability to apply knowledge, calculation and technology to solve mathematical problems in engineering.

In order to assess each competence, different measurable indicators are defined.

Learning activities are planned taking into account these competences to define their content and to decide on the resources and methodologies to be used.

The course was first introduced during the 2009-2010 academic year. Students have online access to material, including the proposed learning activities. Such material will be published shortly, in a textbook (Díaz et al, 2010).

In order to describe our e-learning model, we use the notion of didactic contract. According Cazes, Gueudet, Hersant and Vanderbrouk (2006), the didactic contract is the set of rules between teacher and students which are specific to the knowledge taught. Bound by this didactic contract, students know they have to behave in a given situation by acting in a specific way. In our didactical framework, students work online by doing learning activities, which including study of theory, resolution of direct applications exercises and resolution of problems. The work online implies more freedom but more responsibility and a difficult cognitive effort. As a help in the learning process students use technology: digital tools for communication with the teacher and for self-assessment activities and CAS to work in Mathematics. However the final assessment has been done in a face to face test, without technology.

Students can learn by themselves to use the CAS by consulting the material provided to them in specific tutorials or reading comments about the commands, included in the solved problems (see Villa, 2010).

Usually, proper use of a CAS is defined as an indicator of the generic competence *G6: Use of Technology*. However, in our proposal CAS is integrated into the learning process, thus contributing to the development of other competences.

There are several different roles that CAS can play in the learning of Linear Algebra, from carrying out routine mathematical procedures in realistic applications to providing environments to explore actively. Students can use it at an early stage of learning to verify calculations and, later, software can be used at some or all the stages of solving a problem.

2 CHOOSING SOFTWARE

By implementing the new curriculum, adapted to the EHEA, one of the decisions to be made concerned the choice of the appropriate CAS. Considering our e-learning model the following essential requirement have been established:

- Symbolic, numerical and graphical Linear Algebra features
- Accessibility and ease of installation
- Good maintenance
- Wide diffusion

On the other hand, we wish to make a commitment to free software, which seems especially appropriate if we desire to encourage self-learning among students.

As stated in Stallman (1999), the free software offers:

- Freedom to be used anywhere and for any purpose.
- Freedom to study and to adapt it to our needs.
- Freedom to distribute it to students.

Furthermore, open-source software enables the development of open applications, with technical and economic independence and lack of restrictions of use and promotes in students ethical values such as freedom of knowledge and solidarity.

After assessing different options of free mathematical software (see Mora et al, 2009), we have chosen wxMaxima (http://maxima.sourceforge.net).

Maxima has interesting features in Linear Algebra. It is very easy to use and, since it is freely distributed, students can access it easily and gain a wealth of information.

3. DEVELOPING COMPETENCES BY SOLVING PROBLEMS WITH MAXIMA

The aim of this section is to analyze how Maxima help students to develop the desired competences. The use of Maxima has focused on solving problems. Therefore, let us explore its contribution to the acquisition of competences, following the stages of the development of a Linear Algebra problem. We shall take the following statement as an example and perform a follow-up at several stages of resolution, analyzing each of Maxima's contributions to the acquisition of competences.

A surveillance device has access to the images from a security CCTV that focuses on four sides of a building. The device is programmed in such a way that it only shows one of the sides of the building on the screen. After showing the same side for one minute, it may "choose" (see figure 1) to maintain the image from the same camera, with probability a ($0 \le a \le 1$) or may access one of the two contiguous sides of the building, with an equal probability, which would be (1-a)/2. The security guard controlling the device introduces the value of the parameter a.

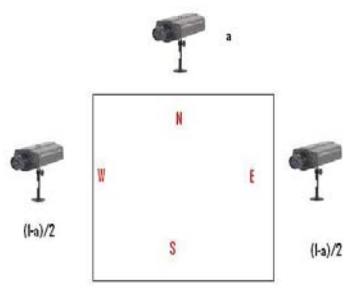

Figure 1: Surveillance device

a) *Which value of parameter should be introduced for displaying the same side of the building constantly?*
b) *At 8:00 a.m. the device displays the North side. The guard introduces the value a = 1/2. Find the probability of showing each of the sides at 9:00 a.m.*

Analyze the same problem with different values of parameter a. Pay special attention to the cases a =0 and a =1.

c) *Study, for different values of a, the behavior of the device, after n minutes (with n very high).*

Below we analyze step by step the stages involved in the resolution of the problem.

Step 1. Modelling

Students must translate the problem, expressed in usual language, to technical language, in order to separate data from aims and choose a model. They must look for similar examples in the required reading literature until the model can be defined.

They will have to identify the data of the problem with mathematical objects. For example, by defining a position vector $V=[x_1,x_2,x_3,x_4]$, that will express the probability that the device will show each of the sides of the building; choosing a notation for the position vector after *n* minutes ($V(n)=[x_1(n),x_2(n),x_3(n),x_4(n)]$), etc.

The starting point at 08:00 a.m is the vector V(0)=[1,0,0,0], indicating that the CCTV is showing the north side of the building with probability 1.

If after *n* minutes the probability vector is *V*(*n*), one minute later it is *V*(*n*+1)=*M*·*V*(*n*), with

$$M=\begin{pmatrix} a & \frac{1-a}{2} & 0 & \frac{1-a}{2} \\ \frac{1-a}{2} & a & \frac{1-a}{2} & 0 \\ 0 & \frac{1-a}{2} & a & \frac{1-a}{2} \\ \frac{1-a}{2} & 0 & \frac{1-a}{2} & a \end{pmatrix}$$

Using Maxima, students can see that the model is the correct one for simple cases. The question in section (a) serves for validating the model. In order for the CCTV always show the same side of the building, the *M* matrix should be the identity; that is, it should be *a* =1.

Once the model and the data have been defined, students must translate reality to a mathematical structure; identify the aim, the procedures and the necessary calculations to attain it. The first step ends when the students have identified the goal: Calculate M^{60} an in general M^n.

In this step the following competences are developed:

G1: Self Learning: Students face the problem alone, without the instructor, although they know that they have the possibility of checking their results with Maxima.

G2: Analysis and Synthesis. For using the computer it is necessary to have the objective well defined.

G4: Communication and writing and G5: Mathematical and Technical writing. In order to communicate the mathematical model to the computer, rigor and precision are necessary. Furthermore, the use of CAS develops a working method that will favour good practices. Perhaps the most striking effect, when expressing the conditions of the problem in formal language, is the rigidity of the program. Students should be strict with both syntax and variable assignment, because if the parameters involved are not well defined, the solution will not be correct.

Step 2. Selecting concepts to be used

Now student must create a scheme of the process to be followed**.** In the reference example, in order to compute efficiently M^n, this scheme includes: Introducing the matrix *M*, finding eigenvalues, eigenvectors and the similarity transformation matrix *P*, computing the inverse matrix P^{-1}, writing the diagonal matrix and using it to compute the powers of *M*. With this work, the specific competence *S1* is developed. Furthermore, making a good scheme is an indicator of *G3*, and writing it correctly is an indicator of *G5*.

Step 3. Resolution

In this stage, students must apply their knowledge of Linear Algebra and carry out with Maxima the pertinent calculations within the solution scheme that have prepared previously. To accomplish this, they must seek and select the most suitable instructions and use them with precision. Upon introducing data in the computer, making appropriate computation, exploring with different values and writing results, the following competencies are developed:

G1: Self Learning: Usually, Maxima offers more than one possibility to achieve the same goal; upon executing the command selected and easily checking the result, the capacity for self-learning is enhanced.

G5: Mathematical and Technical writing: As seen below, when solving the reference problem with Maxima, the interaction between the mathematical and technological formats is seamless, and hence students must be aware of the equivalence between protocols in both formats.

Obviously, in the resolution process the specific competences *S1*, *S2* are developed.

Students can solve the item (b) directly for $a=1/2$ (see figure 2)**.**

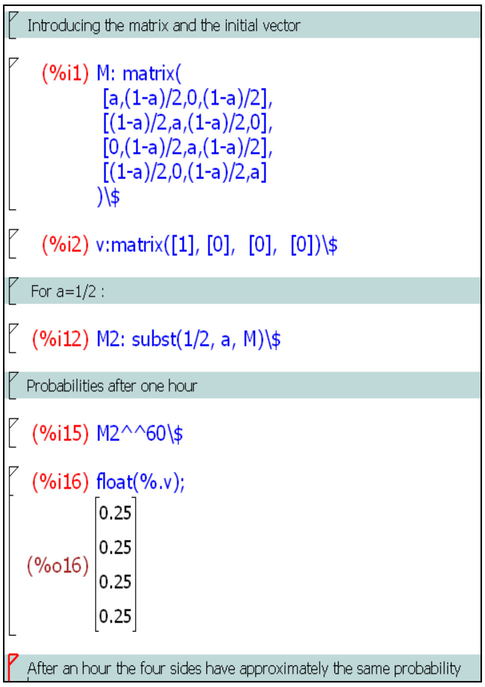

Figure 2: Direct resolution for *a=1/2*

However, despite the calculation power of Maxima it is not possible to perform the same operation for any value of *a*. It is necessary to use Linear Algebra knowledge: as the matrix *M* is symmetric, it is diagonalizable and students must obtain the corresponding diagonal matrix and follow the scheme foreseen in the previous step (see figure 3).

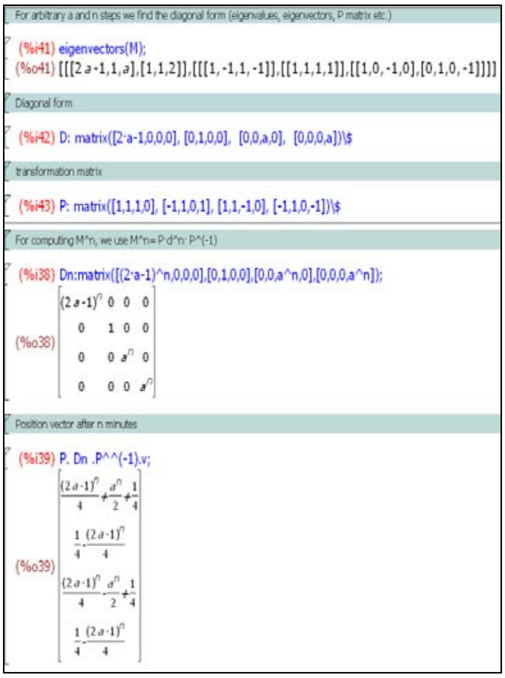

Figure 3: Resolution for any *a*.

By computing $M^n \cdot v$, students can see that for every *a*, with $0 < a < 1$, the position vector tends to [1/4,1/4,1/4,1/4]. For $a = 0$ is $M^n \cdot v = [0,1/2,0,1/2]$ if n is odd and $M^n \cdot v = [1/2,0,1/2,0]$ when n is even. For $a = 1$ is $M^n \cdot v = [1,0,0,0]$.

Step 4: Analysis and interpretation of the outputs

In this step, students must provide an answer to the problem by interpreting the outputs. For example, for any value of *a*, with $0 < a < 1$, the solution obtained means that starting out from any position, after enough time the four sides of the building have almost equal probability of appearing on screen.

Evidently, writing the interpretation is an indicator of competencies *G4* and *G5*. Additionally, in order to express the conclusions correctly it is necessary to order and select the outputs, thus working the competence *G7: Information Management.*

Furthermore, when students analyze application's conditions and different alternatives, they work on the specific competences *S1* and *S2* and also they develop *G8*.

Step 5: Generalizations

Students can generalize results by solving the same problem with different number of sides for the building. For example in hexagonal building results are quite similar. For every *a*, with 0<*a*<1, the probability vector tends to [1/6,1/6,1/6,1/6, 1/6, 1/6]. The case *a=0* and *a=1* are the same behavior. The situation is completely different for a building with an odd or even number of sides. Concretely for a building with an odd number *n* of sides the probability vector tends to [1/*n*, 1/*n*, 1/*n*, ..., 1/*n*]. For a building with an even number *n* of sides the probability vector tends to [2/*n*, 0, 2/*n*, 0, 2/*n*, ...] after an odd number of steps and [0, 2/*n*, 0, 2/*n*, ..] after an even number of steps. By analyzing the different cases students improve the competences *G2*, *G3*, *G7* and *G8*.

4 CONCLUSIONS

When students work independently for solving Linear Algebra problems the proper use of a CAS such as Maxima allowed them to build their own knowledge. This process helps them to develop the competences foreseen.

Thus, in a competence-based e-learning model we propose the following ideas:

- The incorporation of the use of CAS in the learning and assessment processes. For e-learning, free software is advisable.
- Providing students with simple and accessible documentation for consultation and self-learning of CAS handling.
- For each learning goal, exploratory problems and exercises should be proposed to perform with software, including instructions for the use of the CAS.

REFERENCES

Brown, R.B.; McCartney, S. (1995). *Competence is not enough: meta-competence and accounting education*. Accounting Education, v. 4,1, p. 43-53.

Cazes, C.; Gueudet, G.; Hersant, M. and Vandebrouck F. (2006). *Using e-exercise bases in Mathematics: Case studies at University*. Int. Journal of Computers for Math. Learning , 11, p.327-350

Díaz, A.; Hernández, E.; Tejero, L. (2010) *Álgebra para ingenieros* SANZ Y TORRES.

Mora A., Galán. J.L., Aguilera, G., Fernández, A., Mérida, E., Rodríguez, P. (2009*) Scilab and Maxima Enviroment: Towards Free Software in Numerical Analysis*. International Journal for Technology in Math. Education, vol 17, n.2, p.75-79

Mulder, M., Weigel, T. and Collins, K. (2006). *The concept of competence in the development of vocational education and training in selected EU member states. A critical analysis*, Journal of Vocational Educational and Training, 59, 1, p. 65-85.

Stallman, R. M. (1999) *The GNU Operating System and the Free Software Movement*, in DiBo*na*, C. Ockman, S. abd Stone, M. (eds) *Open Sources: Voices from the Open Source Revolution*. O`Reilly& Associates Inc.
http://oreilly.com/catalog/opensources/book/stallman.html

Weinert F.E. (2001). *Concept of competence: A conceptual clarification*, in Ryehen, D., Salganik, L. , *Defining and Selectin Key Competencies*, Hogrefe and Huber, Seattle.

Villa, A. (2010) *Problemas de Álgebra*. CLAGSA.

BIOGRAPHICAL NOTES

Dr. Agustin de la Villa is full professor of Applied Mathematics at Pontificia Comillas and Polytechnic University of Madrid. Dr. Alfonsa García is full professor of Applied Mathematics at Polytechnic University of Madrid. Ana Díaz is full professor of Mathematics for Engineers at the Spanish Open University. Their research interests include the use of technology in Mathematical education, by developing tools to improve the learning process. They have broad experience in the use of CAS as support tools in undergraduate courses of Mathematics and in the design of online learning systems.